Quiz #2, MATH 10 A Section 101 & 105 09/12

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(1 point per item)

① ONLY using the definition of derivative as limit, calculate $f'(x)$ for:

ⓐ $f(x) = 1/x^2$

$f'(x) = \lim_{y\to x} \left(\frac{\frac{1}{y^2} - \frac{1}{x^2}}{y - x} \right) = \lim_{y\to x} \frac{(x^2-y^2)}{(y-x)x^2y^2}$

$= \lim_{y\to x} -\frac{x+y}{x^2y^2} = -\frac{2}{x^3}$

ⓑ $f(x) = 3x^2$

$f'(x) = \lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h} = \lim_{h\to 0} \frac{6xh + 3h^2}{h}$

$= \lim_{h\to 0} 6x + 3h = 6x$

② Calculate the tangent line at $x=0$ for g:

ⓐ $g(x) = (e^x)^3$ $g(0) = 1$

$g'(0) = 3(e^x)^2 e^x \Big|_{x=0} = 3$

$\Rightarrow (y-1) = 3x$

ⓑ $g(x) = \frac{e^{2x+3}}{x^2+3x+5}$ $g(0) = \frac{e^3}{5}$

$g'(0) = \frac{2 e^{2x+3}(x^2+3x+5) - e^{2x+3}(2x+3)}{(x^2+3x+5)^2} \Big|_{x=0} = \frac{(10-3)e^3}{25}$

$\left(y - \frac{e^3}{5}\right) = \frac{7e^3}{25} x$

③ Calculate $g'(x)$ for:

ⓐ $g(x) = e^{(e^x)} \cdot (2^x + x^2)$

Derivatives $e^{e^x} \cdot e^x$ $\ln 2 . 2^x + 2x$

$g'(x) = e^{e^x} \cdot e^x (2^x + x^2) + e^{e^x} (\ln 2 . 2^x + 2x)$

$g'(x) = e^{e^x} (e^x 2^x + e^x x^2 + \ln 2 . 2^x + 2x)$

ⓑ $g(x) = \sin(3^{(x^4)})$

$g'(x) = \cos(3^{x^4}) . \ln 3 . 3^{x^4} . 4x^3$ (Chain rule)

$= 4 \ln 3 \cos(3^{x^4}) . 3^{x^4} . x^3$

③ Calculate the following limits:

ⓐ $\lim_{x\to 4^-} \frac{|x-4|}{x-4}$

If $x < 4$, $|x-4| = -(x-4)$

$\Rightarrow \frac{|x-4|}{x-4} = -1$, then the limit is -1

ⓑ $\lim_{x\to 3} \frac{x^2+1}{8-2^x}$

when $x=3$, $\frac{x^2+1}{8-2^x} = \frac{9}{0}$

$\Rightarrow$ Undefined

ⓒ $\lim_{x\to 0} \frac{\sin(x)}{x}$ (Hint: $\sin'(0) = 1$ and the definition of limit for $\sin'(x)$)

$1 = \sin'(0) = \lim_{h\to 0} \frac{\sin(h) - \sin(0)}{h} = \lim_{h\to 0} \frac{\sin(h)}{h} \Rightarrow$ the limit is 1